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ANNUAL TECHNICAL REPORT

OFFICE OF NAVAL RESEARCH

NONLINEAR WAVES AND INVERSE SCATTERING

BY

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Summary of Research Activities

This has been an extremely active period in our research. We have had seven papers published recently and also have written six preprints which are submitted for publication. The lists are attached. Our research activities are briefly described below.

We are continuing our research in the development of the Inverse Scattering Transform (IST). IST is a method which allows one to solve nonlinear wave equations by solving certain related direct and inverse scattering problems. Research is really two pronged. It is necessary for us to understand and effectively solve both classical and new direct and inverse scattering problems. We use these results to find solutions to nonlinear wave equations much like one uses Fourier analysis for linear problems. Moreover the nonlinear wave equations arise naturally in physical problems. We are particularly interested in fluid dynamical applications.

Although original descoveries employed IST in one spatial dimension, we have developed effective procedures to carry forth the method for multidimensional problems. In one spatial dimension solving the inverse scattering problem requires one to solve a vector Riemann-Hilbert boundary value problem. In multidimensions we have shown that the DBAR method is essential to solve the increase problem. In a special case it reduces to a Riemann-Hilbert problem (sometimes a nonlocal Riemann-Hilbert problem). The method applies to the the multidimensional Schrödinger scattering problem (i.e. the Helmboltz equation) higher order scalar differential operators, multidimensional first order systems and even discrete equations i.e. difference equations. The multidimensional DBAR method is an extremely powerful method to analyze and solve inverse scattering problems. The DBAR method was orignally conceived as part of an earlier grant by ONR to the P.I. We have extended the DBAR method to solve a variety of novel and important inverse scattering problems in multidimensions.

Employing IST we have been able to find new solutions to physically interesting multidimensional nonlinear wave equations. The method requires a proper specification of boundary conditions. Different types of boundary conditions lead to different inverse transforms and different solutions. Important amongst these solutions are coherent structures -- which in multidimensions are rapidly decaying in all directions. In order to understand how the coherent structures arise it is necessary to also study the original governing physical equations and show how the proper specification of boundary values is transferred to the IST solutions. Applications include surface and ocean waves.

We have also been studying a class of physically important forced nonlinear wave equations. In this case the solution structure is rather complicated -- but nevertheless can be developed by the IST method. Applications include the evolution of long nonlinear waves in the presence of moving pressure distributions. Other natural applications to consider are problems of wind-surface wave interactions.

Finally, although the intention of this research program is primarily analytical, we have very interesting results which have application to numerical computation. Specifically we have shown that "standard" numerical schemes frequently have solutions which exhibit spurious chaos. Moreover we have used the IST method to construct new numerical schemes which remain computationally accurate for long times. In order to obtain these results it is necessary for us to solve certain discrete inverse scattering problems. The direct and inverse scattering of difference equations is an extremely important area of study on its own. We are continuing our studies, by analytical methods, of discrete inverse scattering problems in both one and multdimensions.

Recent Publications

1. Nonlinear Evolution Equations, Inverse Scattering and Cellular Automata, M.J. Ablowitz, Solitons in Physics, Mathematics and Nonlinear Optics, Springer-Verlag IMA† Series, Vol. 25, (1990), 1-25.
2. Painlevé Equations and the Inverse Scattering and Inverse Monodromy Transforms, M.J. Ablowitz, Solitons in Physics, Mathematics and Nonlinear Optics, Institute of Math and its Applications, Springer-Verlag IMA† Series, Vol. 25, (1990), 28-43.
3. Forced Nonlinear Evolution Equations and the Inverse Scattering Transform, A.S. Fokas and M.J. Ablowitz, Stud. in Appl. Math., **80** (1989), 253-272.
4. Numerically Induced Chaos in the Nonlinear Schrödinger Equation, B. Herbst and M.J. Ablowitz, Phys. Rev. Lett., **62** (1989), 2065-2069.
5. On Numerical Chaos in the Nonlinear Schrödinger Equations, B. Herbst and M.J. Ablowitz, Integrable Systems and Applications, Orleon, France, Lect. Notes in Physics, **Vol. 342**, Springer-Verlag (1990), 192-206.
6. On the Boundary Conditions of the Davey-Stewartson Equation, by M.J. Ablowitz, S.V. Manakov and C.L. Schultz, Phys. Lett. A., **148** (1990), 50-52.
7. On Homoclinic Structure and Numerically Induced Chaos for the Nonlinear Schrödinger Equation, M.J. Ablowitz and B.M. Herbst, SIAM J. Appl. Math., **50** (1990), 339-351.

IMA†: Institute for Mathematics and Applications

STATEMENT "A" Per Dr.John Lavery
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Preprints

1. Solitons, Numerical Chaos and Cellular Automata, M.J. Ablowitz, B.M. Herbst, J.M. Keiser, PAM† #8 (January 1990) to appear, New Directions in Integrable Systems, World Scientific, Ed. B. Kuperschmidt.
2. On Homoclinic Boundaries in the Nonlinear Schrödinger Equation, by M.J. Ablowitz and B.M. Herbst, PAM† #7, to be published, Proc. of Workshop on Complete Integrability and Inverse Scattering, Montreal (October 1990).
3. On the Complete Integrability of Certain Nonlinear Evolution Equations in One and Two Spatial Dimensions, by M.J. Ablowitz and Javier Villaroel, PAM† #11, to be published, Proceedings: Workshop on Chaos and Order, Canberra, Australia (February 1990).
4. Mel'nikov Analysis and Numerically Induced Chaos by B.M. Herbst and M.J. Ablowitz, PAM† #19, to be published Proceedings: conference Chaos in Australia (February 1990).
5. On the Kadomtsev Petviashvili Equation and Associated Constraints by Mark J. Ablowitz and Javier Villaroel, PAM† #54 (July 1990).
6. On the Hamiltonian formailsm for the Davey-Stewartson System by Javier Villaroel and Mark J. Ablowitz, PAM† #55 (July 1990).

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